## **Exponentially Weighted Averages (with/without bias correction):**

为了理解这个概念，我们可以用一个气温的例子引入，记录了全年每一天的天气气温信息：

图表, 散点图

描述已自动生成

如果我们想观察到气温在这年中的一个动态平均的总体趋势，我们可以使用exponential weighted average：

以上是exponential weighted average的基础表达式，其中 可以被看作是 天气温的一个近似动态平均值，这意味着参数决定了我们观测的窗口大小（约等于弱化了当前的值对于 的影响）。举个例子，如果 那么 就可以被看作是时间点以及之前10天的温度的一个近似均值 (approximately average over 10 days). 下图中的红线对应着，绿线对应。

图表, 散点图

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**In a word, small means more susceptible to outlier but adapts more quickly to the latest value changes**

**Example:** Let consider , which is the exponentially weighted average of the temperature on t=100

It is taking the daily temperature multiplying with an exponentially decaying function and then summing it up.

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That is why, with equal to 0.9, a weighted average has a window size of roughly 10.

## **Bias Correction in exponentially weighted average**

Bias correction主要用于解决exponentially weighted average初始阶段，由于 导致approximate value不准确 (偏小)的问题，Bias correction的具体实施细节如下：

(Assume )

Below is the expanded expression of :

will be significantly less than both and , so is not a very good estimation. To make the estimation more accurate, we can add a **bias correction term** to it:

So that,

As , will approach . As a result we can get **the green curve** shown in the figure below (Compared with no bias correction added, **the purple curve**):

图表, 散点图

描述已自动生成

It can be seen that bias correction mainly correct the estimated value at the beginning. With the increase of , the green curve and the purple curve gradually overlapped.